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String-string duality for some black hole type solutions

Klaus Behrndt[†] and Harald Dorn^{**}

*Humboldt-Universität, Institut für Physik
Invalidenstraße 110, 10115 Berlin, Germany*

Abstract

We apply the duality transformation relating the heterotic to the IIA string in 6D to the class of exact string solutions described by the chiral null model and derive explicit formulas for all fields after reduction to 4D. If the model is restricted to asymptotically flat black hole type solutions with well defined mass and charges the purely electric solutions on the heterotic side are mapped to dyonic ones on the IIA side. The mass remains invariant. Before and after the duality transformation the solutions belong to short $N = 4$ SUSY multiplets and saturate the corresponding Bogomol'nyi bounds.

[†]e-mail: behrndt@qft2.physik.hu-berlin.de, Work supported by the DFG and a grant of the DAAD

^{**}e-mail: dorn@qft3.physik.hu-berlin.de

In recent times it has been conjectured that different types of string theory can be related by different types of duality transformations. One example is the string-string duality [1, 2] relating heterotic string theory compactified on T^4 and the type IIA string theory compactified on $K3$ [3]. Both theories live in $D = 6$ and are related to each other by the field transformation

$$\tilde{\Phi} = -\Phi \quad , \quad \tilde{G}_{\hat{M}\hat{N}} = e^{-2\Phi} G_{\hat{M}\hat{N}} \quad , \quad \tilde{H}_{\hat{M}\hat{N}\hat{P}} = e^{-2\Phi} {}^*H_{\hat{M}\hat{N}\hat{P}} \quad , \quad (1)$$

where $G_{\hat{M}\hat{N}}$, Φ and $H_{\hat{M}\hat{N}\hat{P}}$ are the metric, dilaton and torsion in the stringy frame. The canonical metric remains invariant.

As one example it has been shown that the fundamental string solution on the heterotic side becomes an instanton solution on the type IIA side [4]. In this paper we are going to discuss a much more general class of exact string backgrounds. In $D = 6$ this class is the chiral null model [5, 6], which contains the fundamental string and gravitational waves as special examples, and after the reduction to $D = 4$ it represents many known massive and massless black hole solutions as well as Taub-NUT geometries.

First, we will discuss the heterotic side and recall some known facts about this model. In a second step we are looking on the dual theory and discuss the type IIA duality transform of known heterotic black/white hole solutions.

On the heterotic side we start with the $D = 6$ effective action

$$S = \int d^6x \sqrt{G} e^{-2\Phi} \left[R + 4(\partial\Phi)^2 - \frac{1}{12} H^2 \right] . \quad (2)$$

We are assuming that the reduction from $D = 10$ is trivial, i.e. we have set all gauge fields and scalars coming from this reduction to zero and will ignore furthermore this part of the internal space. We are interested in the following solution of the equations of motion [5]

$$\begin{aligned} ds^2 &= -2F(x)du \left[dv - \frac{1}{2}K(x)du + \omega_I dx^I \right] + dx^I dx^I \\ B &= -2F(x)du \wedge [dv + \omega_I dx^I] \quad , \quad e^{2\Phi} = F(x) \quad , \end{aligned} \quad (3)$$

with

$$\partial^2 F^{-1} = \partial^2 K = 0 \quad , \quad \partial^I \partial_{[I} \omega_{J]} = 0 . \quad (4)$$

In the following we call $v = x^0$, $u = x^5$. Indices take the values: latin from 1 to 3, greek from 0 to 3, large latin from 1 to 4 and hatted large latin from 0 to 5. This

model is a generalization of the gravitational waves ($F = 1$) and the fundamental string ($K = 1, 0^1$) background. It possesses unbroken supersymmetries and is exact in the α' expansion.

Assuming that the fields depend only on x^i and that $v = x^0$ becomes the time (u and x^4 are the internal coordinates) we can reduce this theory to $D = 4$ (for details see Ref.[7]²). Then the solution in $D = 4$ in the canonical frame is given by

$$\begin{aligned}
ds_{can}^2 &= -e^{2\phi}(dx^0 + \omega_i dx^i)^2 + e^{-2\phi} dx^i dx^i , \\
\vec{A}_\mu^{(1)} &= -\frac{1}{2}(F\omega_4, 1)e^{4\sigma}F\omega_\mu , \quad \vec{A}_\mu^{(2)} = \frac{1}{2}(\omega_4, -K)e^{4\sigma}F^2\omega_\mu , \\
e^{-4\sigma} &= G_{44}G_{55} - G_{45}^2 = KF - F^2\omega_4^2 , \quad e^{-2\phi} = \sqrt{KF^{-1} - \omega_4^2} , \\
h_{\mu\nu\rho} &= 6e^{4\phi}\omega_{[\mu}\partial_\nu\omega_{\rho]} ,
\end{aligned} \tag{5}$$

with $\omega_\mu = (1, \omega_i)$. The vector field $\vec{A}_\mu^{(1)}$ comes from the metric and $\vec{A}_\mu^{(2)}$ from the antisymmetric tensor, $h_{\mu\nu\rho}$ and ϕ denote the $4D$ antisymmetric tensor and dilaton, respectively. The moduli space of this model is parametrized by the scalar fields of the theory which can be summarized to 3 complex fields [2]

$$\begin{aligned}
S &= a + ie^{-2\phi} , \quad U = \frac{1}{T^*} , \\
T &= b + ie^{-2\sigma} = B_{45} + i\sqrt{G_{44}G_{55} - G_{45}^2} = F\omega_4 + i\sqrt{KF - F^2\omega_4^2} ,
\end{aligned} \tag{6}$$

where the axion a is defined by

$$h_{\mu\nu\lambda} = e^{2\phi}\sqrt{-g} \epsilon_{\mu\nu\lambda\rho} g^{\rho\tau} \partial_\tau a . \tag{7}$$

Finally we can combine the gauge fields to one graviphoton ($\vec{A}^{(+)}$) and two gauge fields ($\vec{A}^{(-)}$) sitting in the vector multiplet by

$$\begin{aligned}
\vec{A}_\mu^{(+)} &= \frac{1}{\sqrt{2}}(\vec{A}_\mu^{(1)} + \vec{A}_\mu^{(2)}) = -\frac{1}{2\sqrt{2}}(0, 1 + |T|^2) e^{4\sigma}F\omega_\mu , \\
\vec{A}_\mu^{(-)} &= \frac{1}{\sqrt{2}}(\vec{A}_\mu^{(1)} - \vec{A}_\mu^{(2)}) = -\frac{1}{2\sqrt{2}}(2b, 1 - |T|^2) e^{4\sigma}F\omega_\mu .
\end{aligned} \tag{8}$$

Before we go to the dual theory we summarize some properties of this heterotic solution. First, we see that the solution is completely determined by 4 harmonic functions: F^{-1} ,

¹Vanishing K is possible here, but it is singular after reduction to $D = 4$.

²We use the conventions of [7] applied to the reduction from 6 to 4 dimensions. We denote by $\vec{A}_\mu^{(1)}$ and $\vec{A}_\mu^{(2)}$ the internal space vectors $(A_\mu^{(1)}, A_\mu^{(2)})$ and $(A_\mu^{(1+2)}, A_\mu^{(2+2)})$, respectively.

K , ω_4 and a or equivalently by the two complex scalars S and T . Let us look on some special examples. We choose the following harmonic functions ($r^2 = x^i x^i$, $R^2 = x_1^2 + x_2^2 + (x_3 - i\alpha)^2$)

$$\begin{aligned} K &= \text{Re} \left(1 + \frac{2m}{R} \right), \quad F^{-1} = \text{Re} \left(1 + \frac{2\bar{m}}{R} \right), \\ \omega_4 &= \text{Re} \left(\frac{2q}{R} \right), \quad a = \text{Im} \left(\frac{2n}{R} \right). \end{aligned} \quad (9)$$

They describe a rotating black hole, see e.g. [6] and refs. therein. The corresponding mass, electric and magnetic charges ($g_{00}^{can} = -1 + \frac{2M}{r}$, $F_{0r}^{(\pm)} = \frac{\vec{Q}_{(\pm)}}{r^2}$, $*F_{0r}^{(\pm)} = \frac{\vec{P}_{(\pm)}}{r^2}$ for large r) are given by

$$M = \frac{1}{2}(m + \bar{m}) \quad , \quad \vec{Q}_{(+)} = \frac{1}{\sqrt{2}}(0, 2M) \quad , \quad \vec{Q}_{(-)} = \frac{1}{\sqrt{2}}(-2q, m - \bar{m}) \quad . \quad (10)$$

Note there are no magnetic charges. The Bogomol'nyi bound contains only the (+) sector and is saturated $|Q_{(+)}|^2 = 2M^2$. Three cases are of special interest [6, 10]

1.) *massless black/white holes*, $\bar{m} = -m$

In this case the graviphoton sector is uncharged. These configuration have an additional naked singularity at $r_0^2 = 2|Q^{(-)}|^2$ which is repulsive [8] and cannot be obtained as an extremal limit of known black hole solutions. They can be identified as string states with $N_R = \frac{1}{2}$, $N_L = 0$.

2.) *Kaluza-Klein black holes*, $\bar{m}m = 0$

These solutions are characterized either by a constant F or constant K , i.e. they correspond to the fundamental string or gravitational waves in original $D = 6$ theory. The string states are $N_R = \frac{1}{2}$, $N_L = 1$.

3.) *extremal dilaton black holes, flat internal space*

The flat internal space corresponds to $b = 0$ and $|T| = 1$ ($q = 0$, $\bar{m} = m$). In this case the T modulus is trivial and we have only the graviphoton, i.e. in this case we have pure gravity. The string states are given here by a discrete set with $N_R = \frac{1}{2}$, $N_L > 1$.

Of course, these are only some cases and the general solution contains many others, but these solutions found special interest in the literature. At the end we will return to these cases and discuss their dual partners (type IIA).

In the next step, we perform the string/string duality in $D = 6$ and reduce the dual model to $D = 4$. The duality transformation as defined in (1) leads to

$$\begin{aligned} d\tilde{s}^2 &= -2du[dv - \frac{1}{2}Kdu + \omega_I dx^I] + F^{-1}dx^I dx^I, \\ e^{2\tilde{\Phi}} &= F^{-1} \end{aligned} \quad (11)$$

immediately. A little bit more effort is needed to find the dual \tilde{B} , \tilde{H} . We start with

$$H_{\hat{M}\hat{N}\hat{P}} = 3\partial_{[\hat{M}}B_{\hat{N}\hat{P}]} \quad (12)$$

and find

$$\tilde{H}_{ABC} = -\epsilon^{ABCI}\partial_I F^{-1} , \quad \tilde{H}_{AB5} = -\epsilon^{ABIJ}\partial_I\omega_J . \quad (13)$$

All other components are zero.

As a side remark we would like to emphasize that the special case $K = \omega_I = 0$ just gives the axionic soliton background [9]. Hence, we get the known statement that the elementary (or fundamental) string background on the heterotic side is mapped to a soliton on the type II side [4]³

$$d\tilde{s}^2 = -2dudv + e^{2\tilde{\Phi}}dx^I dx^I , \quad \tilde{H}_{ABC} = -\epsilon^{ABCI}\partial_I e^{2\tilde{\Phi}} . \quad (14)$$

Our background is then a natural generalization of this dual pair. We should also mention that $\omega_I = 0$, $K \neq 0$, $F \neq 0$ has been generalized to chiral null models with curved transverse space [14].

To write down \tilde{B} corresponding to (13) we introduce the following transformations (assuming $\partial_4 = 0$)

$$\begin{aligned} \partial_a\omega_4 &= \epsilon^{abc}\partial_b\tilde{\omega}_c , & \partial_{[a}\omega_{b]} &= \frac{1}{2}\epsilon^{abc}\partial_c\tilde{\omega}_4 , \\ \partial_a F^{-1} &= -\epsilon^{abc}\partial_b\bar{\omega}_c . \end{aligned} \quad (15)$$

This system of differential equations for the new quantities $\tilde{\omega}_I$, $\bar{\omega}_i$ is soluble on shell. Note that via (5),(7) $a = -\tilde{\omega}_4$ has been introduced as the axion already. Using these new functions we get for the nonzero components finally

$$\tilde{B}_{m4} = -\bar{\omega}_m, \quad \tilde{B}_{m5} = -\tilde{\omega}_m, \quad \tilde{B}_{45} = -\tilde{\omega}_4. \quad (16)$$

Now we repeat the dimensional reduction on the dual side and get

$$\begin{aligned} d\tilde{s}_{can}^2 &= ds_{can}^2, \quad \tilde{\phi} = \sigma, \quad \tilde{\sigma} = \phi, \\ \tilde{A}_{\mu}^{(1)} &= -\frac{1}{2}(\omega_4, F^{-1})\omega_{\mu}e^{4\tilde{\sigma}} = \tilde{A}_{\mu}^{(1)}, \\ \tilde{A}_{\mu}^{(2)} &= \frac{1}{2}(\bar{\omega}_{\mu}, \tilde{\omega}_{\mu}) - \frac{1}{2}(F^{-1}, -\omega_4)\tilde{\omega}_4\omega_{\mu}e^{4\tilde{\sigma}}. \end{aligned} \quad (17)$$

³For the inclusion of an additional v -dependence see [13].

To unify the notation we have introduced $\bar{\omega}_0 = \tilde{\omega}_0 = 0$. In contrast to the original case the 4D antisymmetric tensor turns out to be nonzero

$$\tilde{b}_{\mu\nu} = \omega_4 F^2 e^{4\tilde{\phi}} \omega_{[\mu} \bar{\omega}_{\nu]} + F e^{4\tilde{\phi}} \omega_{[\mu} \tilde{\omega}_{\nu]}. \quad (18)$$

Taking into account also the gauge field contribution [7] the related field strength \tilde{h} is

$$\begin{aligned} \tilde{h}_{mnl} &= -e^{4\tilde{\phi}} \epsilon^{mnl} \omega_k \partial_k (F \omega_4), \\ \tilde{h}_{0nl} &= -e^{4\tilde{\phi}} \epsilon^{nlk} \partial_k (F \omega_4). \end{aligned} \quad (19)$$

From these equations we can read off the dual axion

$$\tilde{a} = F \omega_4 = b. \quad (20)$$

Thus, as expected the S and T moduli interchange under string-string duality

$$\tilde{S} = T, \quad \tilde{T} = S, \quad \tilde{U} = U. \quad (21)$$

For the special type of solutions introduced in (9) we find in spheroidal coordinates r, θ, ϕ

$$\begin{aligned} \omega_\phi &= -\frac{2nr\alpha \sin^2 \theta}{r^2 + \alpha^2 \cos^2 \theta}, \\ \tilde{\omega}_\phi &= \frac{2q(r^2 + \alpha^2) \cos \theta}{r^2 + \alpha^2 \cos^2 \theta}, \quad \bar{\omega}_\phi = -\frac{2\bar{m}(r^2 + \alpha^2) \cos \theta}{r^2 + \alpha^2 \cos^2 \theta}. \end{aligned} \quad (22)$$

Studying the asymptotic behaviour for large r we can read off the mass, electric and *magnetic* charges

$$\begin{aligned} \tilde{M} &= M, \\ \tilde{Q}_{(\pm)} &= \frac{1}{\sqrt{2}}(-q, m), \quad \tilde{P}_{(\pm)} = \pm \frac{1}{\sqrt{2}}(-\bar{m}, q). \end{aligned} \quad (23)$$

From the technical point of view the appearance of magnetic charges in the dual case is due to the $O(1)$ asymptotic behaviour of $\tilde{\omega}_\phi$ and $\bar{\omega}_\phi$ compared to the fall off for ω_ϕ . We expect the values of q, m, \bar{m} to be restricted by the dyon quantization condition in such a way that $q^2 \pm m\bar{m}, 2qm, 2q\bar{m}$ are integers.

Furthermore, since now electric and magnetic charges are present we have to consider the Bogomol'nyi bound in its generalized forms [11, 12, 2]. For the type IIA case both sectors (\pm) are involved [2]

$$\begin{aligned} \tilde{M}^2 &\geq \text{Max} \left(|\tilde{Z}_1|^2, |\tilde{Z}_2|^2 \right), \\ |\tilde{Z}_{1/2}|^2 &= \frac{1}{2} \left[(\tilde{Q}_1^{(\pm)} \pm \tilde{P}_2^{(\pm)})^2 + (\tilde{Q}_2^{(\pm)} \mp \tilde{P}_1^{(\pm)})^2 \right]. \end{aligned} \quad (24)$$

Z_1 and Z_2 denote the two central charges in the $N = 4$ SUSY algebra. With (23) we get

$$|\tilde{Z}_1|^2 = |\tilde{Z}_2|^2 = \tilde{M}^2 . \quad (25)$$

Like the original heterotic solution our transformed IIA solution belongs to a short multiplet. Again the Bogomoln'yi bound is saturated. Concerning the identification of string states as usual only pure electric solutions can be related to elementary string states. Nonvanishing magnetic charges have to be attributed to solitonic states. In our case $\tilde{\vec{P}} = 0$ implies Kaluza-Klein black holes with $\tilde{M} = \frac{1}{2}m$, $\tilde{\vec{Q}} = (0, \frac{1}{2}m)$. According to the type IIA mass formula this means $\tilde{N}_L = \tilde{N}_R = \frac{1}{2}$ [2]. With respect to the three special cases discussed on the heterotic side we observe that both the transforms of the massless case as well as the dilatonic black hole have magnetic charge vectors orthogonal to the electric ones.

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